FLOW ANALYSIS OF VISCOELASTIC LIQUID FILM IN A ROTATIONAL MIXER

A. V. Kosterin, A. A. Aleksandrovskii, and V. F. Sharafutdinov

UDC 532.135

The flow of a viscoelastic liquid film in a field of centrifugal forces was studied within the framework of the Oldroyd model.

§1. Let us examine the problem of flow of an anomalously viscous liquid along the inner surface of a rapidly rotating cone (Fig. 1) which is rigidly connected to a reading system.

Experiments show that the steady flow of compositions based on certain polymeric materials is not always possible. It is highly probable that this fact is associated with the elastic properties of the medium. It is expedient in the analysis of the indicated slow flow to make use of a differential type rheological model [1]. One of the simplest models of this type that describes various media is the Oldroyd model [2]. It has a form

$$\sigma_{ik} + \lambda_1 \frac{\delta \sigma_{ik}}{\delta t} = 2\mu \left(D_{ik} + \lambda_2 \frac{\delta D_{ik}}{\delta t} \right), \tag{1}$$

where σ_{ik} and D_{ik} are the components of the stress deviator and of the deformation rate tensor, respectively, while $\delta/\delta t$ is the tensor operator of the derivative with respect to time.

Various such operators are known. The most successful is the Jaumann operator because it converts a symmetrical tensor to antisymmetrical [3].

Thus, suppose

$$\frac{\delta \sigma_{ik}}{\delta t} = \frac{\partial \sigma_{ik}}{\partial t} + V^j \frac{\partial \sigma_{ik}}{\partial x^j} - \omega_{km} \sigma_i^m - \omega_{im} \sigma_k^m, \tag{2}$$

where V^j and $\omega_{km} = {}^1/_2(\nabla_m V_k - \nabla_k V_m)$ are components of the velocity vector and of the antisymmetrical part of the velocity gradient, respectively.

The use of the tensor derivative with respect to time is necessitated by the presence of large deformations in the indicated blow.

§2. The characteristic features of the problem can be naturally divided into two groups. The first is associated with the properties of the medium: $1 \gg \lambda_1 > \lambda_2 \geq 0$, and the viscosity μ is sufficiently high. The second is stipulated by the regime of the process [4]: $V_1 \gg V_2$, $V_3 \approx 0$, and $\partial/\partial x^3 \approx 0$.

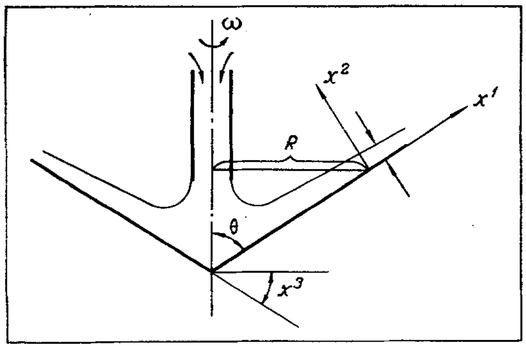

Fig. 1. Schematic representation of liquid flow in a conical rotor.

V. I. Ul'yanov-Lenin Kazan' State University. S. M. Kirov Kazan' Chemical Technology Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 34, No. 2, pp. 249-252, February, 1978. Original article submitted January 10, 1977.

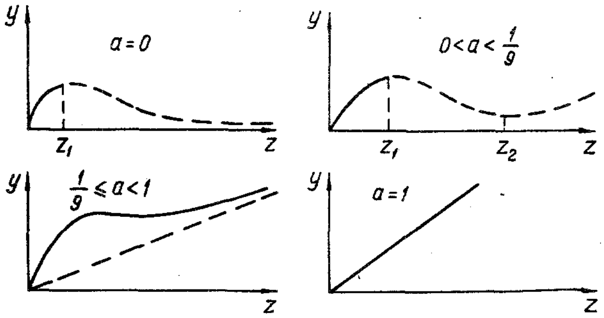

Fig. 2. Rheology at various values of parameter a.

Let V be the characteristic projection of velocity on the generatrix, and L is the characteristic size of the cone. Let us suppose that

$$\frac{\lambda_1 V}{L} \ll 1. \tag{3}$$

The indicated features of the problem together with the definition (3) permit the simplification of Eq. (1). In order to do this, it is sufficient to evaluate the orders of magnitudes in this equation. As a result, for the steady state we will have

$$\sigma_{ik} - \lambda_1 (\omega_{km}\sigma_i^m + \omega_{im}\sigma_k^m) = 2\mu [D_{ik} - \lambda_2 (\omega_{km} D_i^m + \omega_{im} D_k^m)]. \tag{4}$$

Here and further on, the physical components of tensors play a part. Inasmuch as $\omega_{i3} = -\omega_{3i} = 0$, the relationship (4) is equivalent to the following system of equalities:

$$\begin{cases} \sigma_{11} - 2\lambda_1\omega_{12}\sigma_{12} = 2\mu (D_{11} - 2\lambda_2\omega_{12}D_{12}), \\ \sigma_{22} + 2\lambda_1\omega_{12}\sigma_{12} = 2\mu (D_{22} + 2\lambda_2\omega_{12}D_{12}), \\ \sigma_{12} + \lambda_1\omega_{12} (\sigma_{11} - \sigma_{22}) = 2\mu [D_{12} + \lambda_2\omega_{12} (D_{11} - D_{22})]. \end{cases}$$

Let us now subtract the second equality from the first and substitute into the third. We then obtain

$$\sigma_{12} (1 + 4\lambda_1^2 \omega_{12}^2) = 2\mu D_{12} (1 + 4\lambda_1\lambda_2\omega_{12}^2) + 2\mu\omega_{12} (\lambda_2 - \lambda_1) (D_{11} - D_{22}).$$

In this equality the values ω_{12} and D_{12} are of the same order, and $|(\lambda_2 - \lambda_1)(D_{11} - D_{22})| \ll 1$ by virtue of (3). Finally, therefore, we will have

$$y = \frac{z + az^3}{1 + z^2}, \tag{5}$$

where

$$y = \lambda_1\sigma_{12}/\mu, \quad z = \lambda_1 \frac{\partial V_1}{\partial x^2} \geqslant 0, \quad a = \lambda_2/\lambda_1.$$

§3. Let us analyze the function (5). We have

$$\begin{cases} y_z' = (1 + z^2)^{-2} [az^4 - (1 - 3a) z^2 + 1], \\ y_{zz}'' = (1 + z^2)^{-3} 2z (1 - a) (z^2 - 3). \end{cases} \tag{6}$$

A simple investigation reveals that at $0 \le a < 1/9$ a real solution z_1 of equation $y_z' = 0$ exists, whereupon $y_{zz}''(z_1) < 0$ (Fig. 2). The effective viscosity at point z_1 becomes zero, and at certain $z > z_1$ it is negative, which has no physical meaning. The steady state is probably disrupted at this point.

Thus, at $0 \le \lambda_2/\lambda_1 < 1/9$ the Oldroyd model describes steady state only when $0 \le z \le z_1$.

Furthermore, if $1/9 \le a < 1$, then the function y(z) describes the entire Oswald curve [5]. Let us note that the values 0 and 1 correspond to the Maxwell model and the Newtonian fluid, respectively.

§4. Let us find the velocity distribution and the equation for determining film thickness. For simplicity's sake, let us examine the case $a = 0$.

The equations for motion under stress for the examined problem are easily integrated [4]. In particular,

$$\sigma_{12}=\rho F_1(\delta_0-x^2), \tag{7}$$

where $F_1 = \omega^2 R\sin\theta - g\cos\theta$.

From (5) and (7) it follows that

$$\frac{\partial V_1}{\partial x^2}=\frac{1}{\lambda_1}\left[\frac{A}{\delta_0-x^2}-\sqrt{\left(\frac{A}{\delta_0-x^2}\right)^2-1}\right], \tag{8}$$

where $A = \mu/2\lambda_1\rho F_1$.

Further, upon integrating (8) taking the limiting condition $V_1(x^1, 0) = 0$ into account, we will have

$$V_1=\frac{A}{\lambda_1}\left\{\ln\left[1+\sqrt{1-\left(\frac{\delta_0}{A}\right)^2}\right]-\ln\left[1+\sqrt{1-\left(\frac{\delta_0-x^2}{A}\right)^2}\right]+\sqrt{1-\left(\frac{\delta_0-x^2}{A}\right)^2}-\sqrt{1-\left(\frac{\delta_0}{A}\right)^2}\right\}.$$

We find the film thickness δ_0 from the condition of constant delivery

$$2\pi\int_0^{\delta_0} RV_1dx^2=q,$$

or

$$A^2\left\{\arcsin\sqrt{1-\left(\frac{\delta_0}{A}\right)^2}+\frac{1}{2}\arcsin\left(\frac{\delta_0}{A}\right)-\frac{1}{2}\left(\frac{\delta_0}{A}\right)^2\sqrt{1-\left(\frac{\delta_0}{A}\right)^2}+\left(\frac{\delta_0}{A}\right)^2-\frac{\pi}{2}\right\}=\frac{\lambda_1 q}{2\pi R}. \tag{9}$$

It is interesting to find the value of R_{max} below which it is possible to have steady flow.

From (5)-(7) we find that $\sigma_{12max} = \mu/2\lambda_1$, $\delta_0/A = 1$, and from (9) we obtain

$$R_{max}=\frac{\pi(4-\pi)\mu^2}{8\rho^2\lambda_1^3\omega^4 q\sin^2\theta}.$$

NOTATION

σ	is the stress tensor deviator;
D	is the deformation rate tensor;
λ_1 and λ_2	are the relaxation times;
μ	is the viscosity coefficient;
V^j	are the velocity components;
δ_0	is the film thickness;
θ	is the half-cone angle;
R	is the distance from the axis to cone generatrix.

LITERATURE CITED

1. C. Truesdell (editor), Continuum Mechanics: Vol. 2, Rational Mechanics of Materials, Gordon and Breach (1965).
2. Rheology (Theory and Supplement) [Russian translation], IL, Moscow (1962).
3. L. I. Sedov, "Understanding various rates of tensor changes," Prikl. Mat. Mekh., 24, No. 3 (1960).
4. Transactions of the Seminar on the Regional Problems [in Russian], KU, Kazan', No. 12 (1975).
5. M. Reiner, Rheology [in Russian], Moscow (1965).